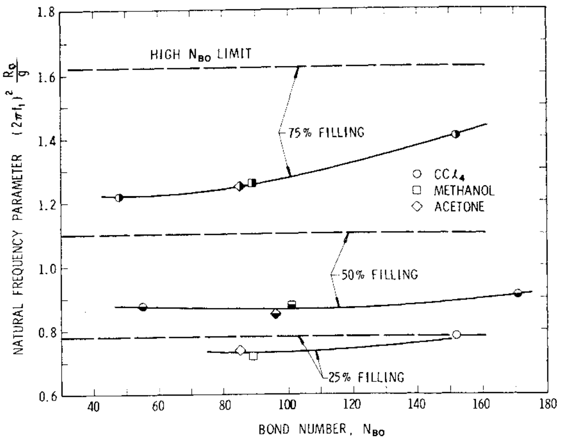

Fig. 6 Variation of natural frequency with Bond number for 1.99/1.0 ellipsoidal tanks.

the 84.4% filling. Similar results hold for large N_{BO} sloshing but, in that case, γ_s for the 15.6 and 84.4% filling are about equal. Thus, once again, the effects of the differences in free surface curvature are evident. Finally, γ_s increases when either N_{GA} or N_{BO} decreases, but an accurate correlation has not yet been obtained.

Ellipsoidal tanks

The N_{BO} range covered in the tests with the oblate ellipsoidal tanks was 48 to 172, based on the free-surface diameter. Liquid filling levels of 25, 50, and 75% of the tank volume were tested; these levels correspond to average liquid depths of 33, 50, and 67% of the minor diameter.

The frequency parameters for the $(2)^{1/2}/1.0$ tank are shown in Fig. 5, and for the 1.99/1.0 tank in Fig. 6. (R_o used in these figures is one-half the major diameter.) The trends shown are the same observed previously in Fig. 4 for spherical tanks, except that here the difference in free-surface curvature between the low and the high filling level is not so great because of the smaller difference in filling levels. There are no existing low-g slosh theories for ellipsoidal tanks of these eccentricities. Damping results are qualitatively similar to those described for spherical tanks.

Conclusions

The method of ultrasmall model simulation of low-gravity propellant sloshing has been shown to give useful data for the prototype N_{BO}'s greater than about ten. The test results show that N_{BO} influences the slosh natural frequency only slightly for cylindrical tanks, but markedly for spherical and ellipsoidal tanks that are over half full. The slosh damping has also been shown to depend upon N_{BO}. Finally, qualitative determinations of the "slosh mass" in an equivalent mechanical model indicate that the amount of liquid participating in the sloshing motion is less than for the corresponding large N_{BO}, hat interface case.

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Use of a Polynomial Force Model to Supplement a Reduced Gravity Model

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In a previous publication[1] the authors presented a polynomial force model to supplement the conventional spherical harmonics representation of the terrestrial gravitational field for the determination of near-equatorial, near-synchronous, near-circular satellite orbits. The present paper extends this analysis to a consideration of the orbit determination accuracy attainable when the polynomial force model is used to supplement a gravity model consisting only of μ and J_2 terms.

Discussion

The approach taken in Ref. 1 was to supplement the conventional spherical harmonics representation of the terrestrial gravity field with a forcing function with (radial, along-track, and cross-track) components represented by polynomials p_i of the form

$$p_i = a_{i1} + a_{i2}(r - r_0) + a_{i3}(\phi - \phi) + a_{i4}(\lambda - \lambda_0)$$

where ϕ = vehicular sublatitude, λ = vehicular sublongitude, r = vehicular separation from the geocenter, i = 1, 3 for radial, in-track and cross-track components, a_{ij} = constant coefficients, and 0 = subscript notation for nominal value. One then solves for the a_{ij} in the same manner as one determines the spherical harmonics of ever increasing degree in a conventional representation of the earth's gravity field.

The "data" utilized were generated from a simulation using an eighth-degree, eighth-order (8/8) model of the earth's gravity field. The "measurements" were taken over a duration of 20 days and consisted of nonsynoptic range measurements originating from tracking stations located in Calif., New England, and Hawaii supplemented by angular "data" originating from eight other stations. The range data were taken at a sampling rate of one observation every 8 sec over 5-min intervals, these latter intervals being separated by 10 hr. The angular data were sampled at a rate of one observation every 24 hr. One-sigma tracking errors were assumed to be 50 ft due to bias and 60 ft due to noise in the range measurements, and 2 arcsec due to noise and bias in the angular measurements. These latter values are considered

Table 1 Optimistic and pessimistic estimates of gravity model errors

Coefficient	μ	J_2	J_3	J_4	J_{22}	J_{31}
Optimistic error, %	0.001	0.02	10	10	10	10
Pessimistic error, %	0.01	0.2	100	100	100	100

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Table 2 Ranges of orbit determination errors (ft) with unsupplemented eighth-degree, eighth-order (8/8) gravity model and model of μ and J_2 only (e = 0.05, i = 5.0°)

Gravity model used in fit[a]	Radial		In-track		Cross-track	
	8/8 model	μ, J_2 only	8/8 model	μ, J_2 only	8/8 model	μ, J_2 only
O	−950 to 110	−1,300 to 500	−3,300 to 6,000	−1,900 to 11,800	−2,500 to 2,700	−3,700 to 4,000
O and P_0	−88 to −48	−98 to −26	1,200 to 1,300	1,100 to 1,300	480 to 710	460 to 1,100
P	−9,800 to 1,700	−8,400 to −490	−45,000 to 53,100	−4,700 to 46,000	−31,100 to 34,300	−34,200 to 37,700
P and P_0	−140 to 25	−110 to −27	530 to 1,400	1,100 to 1,400	−60 to 1,400	400 to 1,100

[a] Letters O and P refer to "optimistic" and "pessimistic" gravity models in Table 1; P_0 = polynominal.

reasonable for Baker-Nunn cameras. For all cases considered, nominal station mislocation errors were taken at 150 ft in latitude and longitude and 25 ft in altitude.

The gravity model errors listed in Table 1 were assumed in the "perfect" 8/8 model used in the simulation. These errors appear to span the precision of our present knowledge of the terrestrial gravity field.[2,3] In Table 1, μ is the product of the universal gravitational constant and the mass of the earth, and J_n and J_{nm} are coefficients of the zonal and nonzonal harmonics, respectively, with the n subscript designating the degree and the m subscript the order.

All of the orbits analyzed in Ref. 1 had four osculating elements (semimajor axis, argument of the ascending node, argument of perigee, and time of perigee passage) in common; with the semimajor axis chosen so as to yield the near synchronous condition. Thus, these orbits were completely specified by their orbital inclinations i, and eccentricities, e. It should also be noted that those two elements serve to approximate the latitudinal and longitudinal excursions respectively; the maximum vehicular sublatitude being nearly equal to the inclination and the longitudinal excursion, $\Delta\lambda$, being closely approximated, for small to moderate eccentricities, by[4] $\Delta\lambda = 2e \sin nt$, where n is the mean motion. Thus, for near-synchronous, near-equatorial satellite orbits a maximum longitudinal excursion of approximately $\pm 2e$ rad will be encountered.

One of the orbits analyzed in Ref. 1 (therein designated as the case 3 orbit) was specified by e = 0.05 and i = 5.0°. This analysis extends that of Ref. 1 to a consideration of the orbit determination accuracy attainable when the polynomial force model is used to supplement a gravity model consisting only of μ and J_2 terms. The gravity model errors listed for μ and J_2 in Table 1 (and designated "optimistic" and "pessimistic") were used in the fit. In this case, all other coefficients in the 8/8 gravity model were set equal to zero for the orbit determination. Table 2 compares ranges of orbit determination errors throughout the 20-day data span associated with this orbit as determined by the 8/8 model with those determined by the μ and J_2 only model. For purposes of this analysis, the orbit determination error is defined as the difference between the position vector resulting from the fit, projected forward (within the 20-day data span) utilizing the same gravity model used in the fit and the perfect epoch vector utilized in the simulation projected forward to the time of interest utilizing the perfect 8/8 gavity model in the simulation.

One is struck by the ability of the polynomial representation to compensate for errors induced by even extremely truncated fitting models. In fact, as long as the polynomial force model is used as a supplement, the resulting orbit determination accuracy is insensitive to choice of base model.

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A Semiempirical Study of Pressure Oscillations in LOX Pump Inducers

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THIS analysis is concerned with the pressure oscillations in the liquid oxygen (LOX) pump inducer of the J-2 engine. A distinct frequency band exists for a certain combination of the pump inlet pressures and mixture ratios.[1] To locate this band of oscillations in a plot of inlet pressure P_i vs mixture ratio MR, a semianalytical first-order model with a retarded time mechanism is introduced. It is necessary to evaluate several constants in the formulation from the available experimental results for a certain inducer under consideration. Since the available experimental data show that the variation between the frequency ranges and inlet pressure magnitudes was approximately linear, two attempts are made to depict first-order models. One is oriented principally toward P_i, with MR appearing only as a multiplier whose influence is limited to narrowing or widening the corridor of pressure oscillations. The other model is MR-oriented.

Formulation of Analytical Model (P_i-Oriented)

It is postulated at the outset that the pressure rate of change in the inducer at any instant, t, is proportional to the net contributions from the pressure at the present instant $t = t$ and from the pressure at a previous instant $t = t - t_1$ (later t_1 is set to be C_2/f); these contributions must be appropriately augmented with respect to the nominal, P_i. In mathematical terms, it can be written

$$\frac{dP(t)}{dt} = \frac{C_3}{MR}\left[\frac{P_i - C_i}{P_0} P(t) - \frac{P_i}{P_0} P\left(t - \frac{C_2}{f}\right)\right] \quad (1)$$

where P_0 = characteristic pressure; C_1 = constant pressure term to be determined from the experimental results; $C_2/f = t_1$; C_2 = pure constant to be determined from the experimental results; f = frequency of the pressure oscillations; and C_3 = scaling factor (sec^{-1}). This equation is solved by Laplace

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